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How Close Are Impossible Worlds? A Critique of Brogaard and Salerno's Account of Counterpossibles

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Abstract

Several theorists have been attracted to the idea that in order to account for counterpossibles, i.e. counterfactuals with impossible antecedents, we must appeal to impossible worlds. However, few have attempted to provide a detailed impossible worlds account of counterpossibles. Berit Brogaard and Joe Salerno's "Remarks on Counterpossibles" is one of the few attempts to fill in this theoretical gap. In this article, I critically examine their account. I prove a number of unanticipated implications of their account that end up implying a counterintuitive result. I then examine a suggested revision and point out a surprising implication of the revision.

1. *Introduction: impossible worlds semantics for counterpossibles*

Counterpossibles are counterfactuals with metaphysically impossible antecedents.[1] It is well known that Lewis-Stalnaker semantics of conditionals have an issue with counterpossibles. Roughly speaking, according to Lewis-Stalnaker semantics, "if it were the case that p, it would be the case that q" ($p > q$) is true iff the nearest p worlds are also q worlds. But what happens when there are no possible p worlds? According to Lewis's favored account, counterpossibles are vacuously true, meaning that regardless of the contents of the specific conditional, they will always come out true. I will call this view Positive Vacuism.[2] To many this seems like a counterintuitive result of Lewis-Stalnaker semantics.

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[1] Following the literature, I am focusing on subjunctives. Brogaard and Salerno note that they believe that their theory applies to indicatives as well but they nevertheless remain focused on subjunctives. Matthias Jenny has suggested to me that it may be fruitful to think more about indicative conditionals, because indicatives suggest an epistemic modality and the more interesting and non-weird cases of conditionals with metaphysically impossible antecedents are when we are trying to figure out whether the antecedent is true or not, i.e. when there is an epistemic possibility but metaphysical impossibility.

[2] Note that Brogaard and Salerno use the term Vacuism to refer specifically to Lewis's view according to which counterpossibles are vacuously true. I suggest instead that we call this view Positive Vacuism to make room for other types of Vacuism, which describe counterpossibles as vacuously false (Negative Vacuism) or as lacking a truth value (Neutral Vacuism). I mention the latter two possible

Here are two examples of counterpossibles that strike some authors as counterexamples to Vacuism:

(1) If Hobbes had squared the circle, sick children in the mountains of South America at the time would not have cared. (Nolan 1997, 544)
(2) If intuitionistic logic were the correct logic, then the law of excluded middle would still be unrestrictedly valid. (Brogaard and Salerno 2013, 643)

The first sentence seems intuitively true, but not trivially so. Change the consequent to "mathematicians at the time would not have cared" and the counterfactual becomes intuitively false. The second sentence seems false, and again, non-vacuously. Replace the consequent with its negation and intuitively you get a true sentence. Perhaps most of us couldn't care less what the imaginary world in which Hobbes had squared the circle would have been like. But sometimes, knowing whether a counterpossible is true or false can be important, as in the second example. When evaluating competing logics, we need a way to assess claims like (2). More generally, whenever we wish to evaluate claims in domains in which all true claims are necessary claims, like logic, mathematics, metaphysics and perhaps basic normative claims, we will likely need a way to evaluate counterpossible claims.[3]

An attractive alternative, suggested already by Lewis himself (1973, 24), is to posit impossible worlds in addition to the familiar possible worlds.[4] An impossible worlds account of counterpossibles makes a counterpossible $p > q$ true iff the closest impossible p worlds are also q worlds. A good impossible worlds account of counterpossibles must explain what impossible worlds are, and how the 'closeness' relation operates on such worlds. Brogaard and Salerno's (henceforth B&S) "Remarks on Counterpossibles" (2013) seeks to provide these explanations. Since it is one of the only attempts to do so, and as such it is widely cited, I believe it deserves careful consideration. I will argue that their account fails as is. I will then

theories despite their absence in the literature because I believe they can be motivated. There is some intuitive pull to the thought that counterfactuals ($p > q$) presuppose the existence of worlds in which the antecedent is true (p worlds). (Other philosophers have noted this presupposition in relation to the semantics of counterfactuals. Kai von Fintel (2001) argues that it explains puzzling context shifts in sequences of counterfactuals.) If presupposition failures yield truth gaps or make the proposition false, the same should apply to counterpossibles. Boris Kment (2014, 25, 220) has recently defended a restricted version of Negative Vacuism. According to Kment, all counterfactuals with logically impossible antecedents are false. Note though that Kment believes that there are impossibilities that are not logical impossibilities, and he is not a vacuist with regard to counterpossibles with non-logical impossibilities as antecedent (see also Field 1996, 375–376). I thank Matthias Jenny for the reference to Kment's theory.

[3] For an elaborate list of motivations for impossible worlds semantics, see Nolan (1997) and, more recently, Jenny (2018).

[4] Proponents of impossible worlds semantics for counterpossibles include: Mares (1997), Nolan (1997), Vander Laan (2004), Krakauer (2012), Jago (2015) and Bernstein (2016).

discuss a potential amendment to their account and point out a few surprising and perhaps counterintuitive implications of the amended account. I conclude with some remarks on how B&S's insights may be incorporated in future attempts to analyze counterpossibles.

Here is the plan for the remainder of this paper: In the next section, I present Brogaard and Salerno's semantics for counterpossibles. The following two sections discuss in turn the two factors that compose their account. The first component is shown to have some unanticipated and potentially problematic consequences. The second component is shown to imply a counterintuitive result, which requires an amendment to the account. The amendment in turn is shown to itself have a surprising implication. I conclude in the final section.

2. *Brogaard and Salerno's account*

B&S's preferred Lewisian account of subjunctive conditionals is the following:

> **Lewisian Account of Subjunctives:** A subjunctive, "if it were the case that *p*, it would be the case that *q*", is true just when every closest *p*-world is a *q*-world, where "closest" is read as "most relevantly similar". (p. 641)[5]

Lewis himself included only possible worlds in his theory. However, if this sort of Lewisian account is generally a good account for subjunctives, it is tempting to extend it to accommodate counterpossibles by adding impossible worlds to the theory. For example, it is tempting to think that (1) is true because in the closest (impossible) worlds in which Hobbes succeeds in squaring the circle, sick children in the mountains of South America do not care. And it is tempting to think that (2) is false because in the closest (impossible) worlds in which intuitionistic logic is true, the law of excluded middle is not unrestrictedly valid. In order to make sense of this idea, we need an account of impossible worlds and the closeness relation between impossible worlds and the base world.

An account of impossible worlds is supposed to be a natural extension of an account of possible worlds. Therefore, B&S are right to choose an account for

[5] This formulation implies the controversial limit assumption, that there is at least one closest world. But it is clear from what they say earlier that B&S, like Lewis, reject the limit assumption. Since the limit assumption has no bearing on the discussion in this paper, I will ignore this issue and follow B&S's simpler formulation. A more precise formulation that avoids the limit assumption would have been: A subjunctive "if it were the case that *p*, it would be the case that *q*" is true iff there is a $(p\&q)$ world that is relevantly closer than any $(p\&\neg q)$ world. This is similar to Account 2 in B&S's paper (p. 640) with the addition of the relevance factor that they want.

worlds that can accommodate both possible and impossible worlds. According to their account, a world is (pp. 651–652):

> **World**: A maximal set of sentences.[6]

Which in turn they define as:

> **Maximal Set**: A set *S* is maximal iff for any sentence *p*, either *p* or $\neg p$ (or both), is a member of *S*.

Both possible and impossible worlds are maximal sets of sentences on this account, with only the following difference between the two: Possible worlds are deductively closed, meaning that for any sentence contained in the world, the world contains all of its logical consequences as well. In contrast, impossible worlds need not be deductively closed.[7] As is standard, what it means for a sentence to be true in a world is for the sentence to be a member of the set of sentences that composes the given world.

Since the definition of worlds is rather unrestrictive, i.e. any maximal set of sentences counts as a world no matter how incoherent, the onus remains on the closeness relation to predict which counterpossibles are true and which are false.[8] It is therefore this part of their theory that I will focus on. What determines the closeness of impossible worlds? B&S's proposal involves two factors. They formulate a formal account of closeness as an account for comparing pairs of worlds to the base world as follows:

[6] B&S don't specify the sentences' language. I will assume English, although the arbitrary choice of language adds further complications to their theory. For one, English, like any other language, may lack the vocabulary to answer all questions about a given world and therefore be insufficient in expressive power for an account of worlds. Another problem is that different languages may require different numbers of sentences to express the same propositions and therefore might give different results in B&S's account of closeness, making the choice of English in need of justification. I thank Justin Khoo for bringing this issue to my attention.

[7] B&S say that the only deductively closed impossible world is the absurd world, which contains all sentences. Presumably this is because classically you can deduce anything from an impossibility and they assume classical logic. However, this line of reasoning is questionable. If there are metaphysical impossibilities that are not logical impossibilities, and if only from logical impossibilities can you deduce anything, then there may be metaphysically impossible worlds that are deductively closed. To take B&S's own example, it is not clear that from "water is not H_2O", even if it is metaphysically impossible, you can deduce anything even in classical logic. I owe this point to David Enoch.

[8] To give a sense of how unrestrictive their account of worlds is, consider: There are worlds that include some sentence *p*, and the sentence if *p* then some sentence *q*, and the sentence $\neg q$, and the sentence "classical logic is true of this world". These are of course very incoherent impossible worlds, and precisely for this reason their account for closeness should imply that they are very far from our world.

Closeness: For any two impossible worlds w_1 and w_2, w_1 is closer to the base world[9] than w_2 iff

[Less Inconsistent:][10] w_1 does not contain a greater number of sentences formally inconsistent[11] with the relevant background facts [true of the base world] (held fixed in the context) than w_2 does.
And if w_1 and w_2 contain the same number of sentences formally inconsistent with the relevant background facts (held fixed in the context):

[More Implications:] w_1 preserves a greater number of a priori implications between sentences than w_2 does.[12]

On B&S's account, closeness of impossible worlds is determined by the two factors that I have dubbed 'Less Inconsistent' and 'More Implications'. The first factor tells us that the closest worlds are those in which as much as possible is similar to relevant facts in the base world. This can potentially explain why the closest "Hobbes squared the circle" worlds are those in which sick children in South America at the time care nothing about whether Hobbes succeeded in solving some mathematical puzzle that they do not understand, and that will have no effect on their lives. Therefore (1) is true. The second factor tells us that the closer worlds are those in which more a priori implications are preserved. In the second example, the thought is that it is an a priori implication of intuitionistic logic that the law of excluded middle does not hold. After all, the exclusion of the law of excluded middle is just part of what we mean by "intuitionistic logic", so there is a conceptual a priori implication. This is supposed to explain why the closer "intuitionistic logic" worlds are those in which it is not the case that the law of excluded middle holds and (2) is false.

Before analyzing the view in greater detail, let me make two initial observations. First, B&S's account lacks a uniformity that an account of closeness can be expected to have. B&S restrict their account of closeness to impossible worlds whereas we should expect a general (and hopefully unified) account of closeness for both possible and impossible worlds. After all, in addition to comparing impossible worlds with each other, we need our theory to tell us how impossible

9 Ordinarily, the "base world" will just be the actual world. Sometimes this is not true, as when counterfactuals are embedded within counterfactuals ($p > (q > r)$). I thank Matthias Jenny for correcting me here.

10 For convenience, I added in parentheses the names I will use for the two factors which constitute B&S's account of closeness.

11 By this, they mean formal inconsistency according to classical logic (p. 651).

12 I will discuss their notion of "a priori implications" shortly. They use "a priori*" to mark their relativistic notion of a priority. For simplicity, I will omit the star mark. Nothing will depend on specifics of this notion, and in any case, they don't provide a detailed account of a priority.

worlds stand in relation to possible worlds. In fact, surprising as it may seem at first, several authors have argued that sometimes impossible worlds can be closer than possible worlds (Nolan 1997, 550; Vander Laan 2004, 271).

Second, their method of measuring differences and measuring degrees of a priori implications is by counting sentences. Vander Laan (2004, 272–274) has already warned against this sort of method.[13] One of the features of this method is that it gives equal weight to all relevant differences. In this respect, their account seems to go against Lewis's (1973, 91–95) insight that weights of different standards of comparison can differ depending on context. Surprisingly we will soon learn that their account actually implies that we never need to count, and this is contrary to how B&S themselves present it.

Now let's move to a detailed analysis of B&S's account of closeness.

3. Minimizing discrepancies with the base world

In this section, I will discuss a number of implications of the first factor in B&S's account: Less Inconsistent. A pertinent difficulty in assessing their account is that they do not provide an account for "relevant background facts". This makes their account flexible enough to give in an ad hoc fashion the right predictions to many potential counterexamples. Therefore, in this section I will not be able to provide a conclusive counterexample to their theory. However, I will show that in adjusting the set of relevant background facts so that their theory makes the desired predictions, a number of bullets are bitten.

First, I will prove that their account has the following implication:

> (L1) For any counterpossible of the form $p > q$ and any formally consistent base world (such as the actual world) all the nearest p-worlds must be such that if there is any sentence formally inconsistent with relevant background facts, then it is p (and p alone).

The proof is as follows. For any p and any base world, there is a world identical to the base world, except that it includes p as well. Call this world the *constructed world*. If the base world is formally consistent, then the only sentence in the constructed world that can potentially be formally inconsistent with background facts, i.e. a subset of the base world, is p itself. No p-world can have fewer sentences formally inconsistent with relevant facts than that. Therefore, because of Less

[13] Vander Laan's argument doesn't apply because he considers only a method that counts all discrepancies with the base world, rather than just discrepancies with *relevant* facts in the base world, as B&S do. This context dependence complicates matters. I thank an anonymous reviewer for referring me to Vander Laan's argument.

Inconsistent, all the nearest p worlds are such that if there is any sentence formally inconsistent with relevant background facts, then it is p (and p alone). (Recall that according to B&S's theory, maximizing a priori implications only comes in after we've found the worlds with the least inconsistencies with relevant background facts.)

From (L1) we can derive a further implication of their account:

> (L2) For any counterpossible of the form $p > q$ and any formally consistent base world, if any sentence that formally implies q is a relevant background fact in the context, then $p > q$ is true.

The proof is as follows. We know already from L1 that if the base world is consistent, then all the nearest p-worlds cannot contain any sentence inconsistent with a relevant background fact other than p. Therefore, if any sentence that formally implies q is among the relevant background facts, then the nearest p-worlds cannot contain any sentence formally inconsistent with q, including $\neg q$. Since worlds are maximal, this implies in turn that all the nearest p-worlds in such a context must include q. Therefore, in such a context, $p > q$ must be true.

L2 creates trouble for two of B&S's own examples, assessed by their own stated intuitions about these examples. Here are the examples:

(3) If water had not been H_2O, it would not have been XYZ.
(4) If paraconsistent logic were correct, then the principle of explosion would still be valid.

B&S have the intuition that (3) and (4) are false, and this seems correct.[14] However, whether or not their account gives this prediction will depend on which are the relevant background facts. B&S seem to believe that "water is not XYZ" is a relevant background fact in (3) and that "the principle of explosion is valid" is relevant in (4).[15] However, based on L2 we know that if "water is not XYZ" is a relevant fact to (3), then B&S's theory implies that (3) is true, contrary to their intuition. Similarly, if "the principle of explosion is valid" is a relevant fact to (4), then their theory implies that (4) is true, contrary to their intuition. We can only get

[14] Sentence (4) is discussed explicitly on p. 654. I replaced the Latin with English for the sake of consistency (see note 6). Their judgment of (3) is implied in several places, the most explicit being on p. 653 fn. 16, in which they state that "if water had not been H_2O, it might have been XYZ" is true.

[15] They say on p. 653 that "w_2 expresses a greater number of actually true Russelian propositions than w_1". This can only matter if "water is not XYZ" (or anything that formally implies it) is a relevant fact when assessing "water is not H_2O" counterpossibles. Similarly, on p. 654 they state that "w_1 contains a greater number of relevant actually true sentences" because it contains the sentence "ex falso quodlibet is valid".

the intuitive result if we assume that the aforementioned sentences (and anything that formally implies them) are not among the relevant background facts. This is one bullet they'll have to bite.

From L1 we can derive another implication:

> (L3) For any counterpossible $p > q$ and any formally consistent base world, if $\neg p$ or any sentence that formally implies $\neg p$ is among the relevant background facts, then all the nearest p-worlds will contain an infinite number of sentences formally inconsistent with p, such as $\neg\neg\neg p$ and $\neg\neg\neg\neg\neg p$ and so on.

Here is the proof. If $\neg p$ (or any sentence that formally implies $\neg p$) is among the relevant background facts, then p is inconsistent with a relevant background fact. It follows from L1 that in all the nearest p-worlds, p must be the only sentence inconsistent with the relevant background facts. Therefore, none of the nearest p-worlds include any other sentence formally inconsistent with $\neg p$. Hence, they cannot include $\neg\neg p$ and $\neg\neg\neg\neg p$ and any other sentence which consists in an even number of negations to p. Since worlds are maximal, all the nearest worlds must therefore include the negations of these, e.g. $\neg\neg\neg p$ and $\neg\neg\neg\neg\neg p$ and so on (odd number of negations to p).

If ever $\neg p$ or any sentence that formally implies $\neg p$ is among the relevant background facts, we get an extremely counterintuitive result. An example will help demonstrate the problem. Consider the following counterpossible:

(5) If paraconsistent logic were correct, then paraconsistent logic would not not not be correct.

The sentence is odd, as most counterpossibles tend to be. Nevertheless, if we are favorable to non-vacuism, it seems we should think that (5) is false. However, if "paraconsistent logic is not correct" is a relevant background fact, then B&S's account predicts counterintuitively that (5) is true. In order to avoid such counterexamples, B&S will have to bite another bullet, namely, that $\neg p$ or any sentence that formally implies $\neg p$ can never be among the relevant background facts. And this is contrary to what they sometimes suppose.[16]

You may think that this is a bullet that makes sense to bite. After all, in counterfactual contexts we are interested in what things would be like if the antecedent p were true, setting aside the fact that in the actual world the antecedent is false. However, if we accept this restriction on the set of relevant background facts, we get a surprising result. The surprising result derives from the following implication of B&S's theory:

[16] This conclusion is contrary to B&S's intuition that "water is H_2O" is a relevant background fact when assessing the counterpossible "if water had not been H_2O then it would have been H_2O" (p. 653).

(L4) For any counterpossible $p > q$, and any formally consistent base world, if no sentence that formally implies $\neg p$ is among the relevant background facts, then all the nearest p-worlds include no sentence that is formally inconsistent with any of the relevant background facts.

The proof for L4 is simple. L1 says that in all the nearest p-worlds, p can be the only sentence formally inconsistent with relevant background facts. If the set of relevant background facts includes no sentence formally inconsistent with p, then p itself is formally consistent with the relevant background facts. Hence, there can be no sentence in the nearest p-worlds that is formally inconsistent with the relevant background facts.

L4, in conjunction with the claim from above that $\neg p$ or anything that formally implies $\neg p$ is never among the relevant background facts, implies that there can never be among the nearest p-worlds any world that includes any sentence that is formally inconsistent with any background relevant fact. Is there a problem with this result, other than the fact that B&S probably did not have it in mind? Without an account of relevant background facts, this result is not in itself enough to construct a conclusive counterexample. However, it does have the following two consequences. Less Inconsistent is misleading as it is formulated. We need not count inconsistencies with relevant background facts because the account is intolerant to any inconsistencies with these facts. In addition, once we know that this is what Less Inconsistent implies, we realize how much of the burden of their account falls on defining the relevant set of background facts, which they do not account for, and maximizing a priori implications, to which we now turn.

4. *Maximizing a priori implications*

Let us now examine the second part of the account, namely More Implications. To get a feeling of what B&S mean by a priori implications, consider the following two sentences:

(2) If intuitionistic logic were the correct logic, then the law of excluded middle would still be unrestrictedly valid.

(6) If water had not been H_2O, it would have been H_2O.

Their thought is that (2) and (6) are false because there is a sense in which "water is not H_2O" implies, in some salient sense of "implies", that it is not the case that "water is H_2O", and "intuitionistic logic is correct" implies, in some salient sense of "implies", that it is not the case that "the law of excluded middle is unrestrictedly valid".

What kind of implication is this? Whether you understand logical implication as a matter of necessity, meaning that in any possible world in which the antecedent is true, the consequent is true as well, or as a matter of logical form, meaning that the antecedent implies the consequent in virtue of logical form alone, the implication here is not logical implication. It isn't the former, modal implication, because the antecedent is false in all possible worlds,[17] so "intuitionistic logic is correct" modally implies that the unrestricted law of excluded middle is valid just as much as it modally implies that it is invalid, and (2) should come out true. And it isn't the latter, formal implication, because, at least in (2), the logical relation between the antecedent and consequent does not hold in virtue of logical form. B&S's thought is that, despite the lack of logical implication, in both cases there is something we know a priori to follow from the antecedent. We may not know a priori that water is H_2O (even if it is metaphysically necessary), but we know a priori that if it is not H_2O, then it is not H_2O. Similarly, we know a priori that if intuitionistic logic is true, then the law of excluded middle is not unrestrictedly valid.

B&S formulate a priori implications as follows:

> **A priori Implication**: For a speaker s in a context c, *p* a priori implies *q* iff for s in c, *q* is a relevant a priori consequence of P.

Even though they are not explicit, it is more charitable to their view to suppose that *p* doesn't necessarily represent a single sentence, but rather a set of sentences. They do not say much more about what a priori implications are; they take it as a basic notion that we should grasp by examples of the sort mentioned above. They emphasize that what a proposition implies a priori can be context dependent. The intuitionistic logic example may demonstrate why this is so: There may be contexts in which it is a priori that intuitionistic logic (or some other logic) is false, and then it may not be clear what it implies a priori (everything? nothing?). However, there can be other contexts in which intuitionistic logic is not a priori false, and in such contexts, it a priori implies that the law of excluded middle is false.

While it is attractive to think that a priori implications are relevant for the assessment of counterpossibles and the closeness of worlds, B&S's formalization yields unintuitive results. For instance, their account implies that the following counterpossible is true:

(6) If water had not been H_2O then it might have been H_2O.

I'll explain. B&S's theory has the following implication:

[17] Assuming logical intuitionism is false. If it is true, we can reconstruct an example using a false logic.

(L5) For any world w_1, any world w_2 that is the union of w_1 with *any* set of sentences that are members of the base world, is at least as close as w_1 (in any context).

The proof is simple. Adding to w_2 sentences from the base world cannot add formal inconsistencies with relevant background facts in the base world. Therefore, as far as Less Implications is concerned, any such pair of worlds would be tied regardless of the context (which determines the set of relevant background facts). Next, as far as More Implications is concerned, w_2 cannot include any fewer a priori implications in comparison to w_1. The reason is that an a priori implication is a relation between sentences. Any sentence in w_1 is in w_2 as well. Therefore, any a priori implication in w_1 is included in w_2.

From L5 we can derive the further result that among the closest "water is not H_2O" worlds, there must be a world that includes "water is H_2O". Here's the proof. Suppose there is a world w_3 that is among the closest "water is not H_2O" worlds. There exists a world, call it w_4, that is the union of w_3 with "water is H_2O". L5 implies that w_4 is at least as close as w_3. Since w_3 is among the closest worlds, then so is w_4. w_4 includes "water is H_2O". Therefore, B&S's account counterintuitively implies that (6) is true.

There is no getting around this result without revising the account. One salient possibility is to revise More Implications in the following manner. We want an account that implies that the less coherent a world is, the farther away it is. The way we measure coherence, if we remain in line with B&S's thinking, will have to do with a priori implications. Therefore, what we can say is that the more a world contains sentences formally inconsistent with a priori implications of other sentences, the farther away they are. In the above example, w_4 might[18] now come out farther away than w_3 because w_4 contains the additional sentence "water is H_2O", which is formally inconsistent with "water is not H_2O". Let us then see what happens if we replace More Implications as follows:

Closeness*: For any two impossible worlds w_1 and w_2, w_1 is closer to the base world than w_2 iff

> **Less Inconsistent:** w_1 does not contain a greater number of sentences formally inconsistent with the relevant background facts [true of the base world] (held fixed in the context) than w_2 does.
> And if w_1 and w_2 contain the same number of sentences formally inconsistent with the relevant background facts (held fixed in the context):
> **Less Disimplications**: w_1 includes fewer sentences formally inconsistent with a priori implications of other sentences (in w_1), than w_2 does (with sentences in w_2).

[18] I say 'might' because if w_3 has infinite disimplications (see definition below), then w_4 includes no more than w_3. This issue with infinities will be explained shortly.

As I will now explain, this suggestion leads to another surprising result. To derive the result, I will need a further assumption that is extremely intuitive:

(L6) Every sentence a priori implies itself.

It is difficult to imagine a context in which some sentence p doesn't a priori imply p. The following is a consequence of L6:

(L7) Any world that contains one sentence formally inconsistent with an a priori implication of another sentence in that world, contains infinite sentences formally inconsistent with a priori implications of other sentences within the world.

For suppose some world, w, contains one sentence formally inconsistent with an a priori implication of another sentence in that world (henceforth, a *disimplication*). For instance, suppose p a priori implies $\neg q$, and w includes p and q (or any other sentence formally inconsistent with $\neg q$). Now q (or some other sentence formally inconsistent with $\neg q$) a priori implies itself. Therefore, any sentence which consists of some number of negations of q, must be formally inconsistent with either $\neg q$ or with q, both of which are a priori implications of other sentences in w. Since worlds are maximal, such a world must contain infinitely many disimplications.[19]

L7 implies that there are only two kinds of worlds: worlds with zero disimplications and worlds with infinite disimplications. There exists nothing in between the two extremes. From the previous section we know that for any sentence p, all the nearest p-worlds include all of the relevant background facts and any sentence formally implied by them. Now either there is a world that contains all of these and has zero disimplications or there is no such world. Let's consider what must be the case in each possibility.

If there is no such world, then Less Disimplications falls out of the picture because all worlds with the least inconsistencies with background facts have an equal number of disimplications, namely infinity. In turn, this implies that every single world that is formally consistent with the relevant background facts and includes p, is among the nearest p-worlds.

What all this implies is that the revised account, Closeness*, is equivalent to the following:

[19] Moreover, such worlds must contain uncountably infinite disimplications. That is because for *any* sentence r in a maximal world, the world must also contain either $r\&q$ or $r\&\neg q$, both of which are formally inconsistent with an a priori implication, and there are uncountably infinite sentences in every maximal world. Therefore, distinctions between orders of infinity will not be of any help here.

A counterpossible $p > q$ is true iff one of the following obtains:

(a) q is identical to p.
(b) The relevant background facts formally imply q.
(c) There are worlds that contain p and *all* of the relevant background facts and *no* a priori disimplications between sentences and all such worlds contain q.

Now focus on (c). Let S represent the set of sentences that is the union of p and all of the relevant background facts. Let W_s represent the set of all worlds that include S as a subset and that in addition have no a priori disimplications. What must be the case in order for all of the members of W_s to include q (except for possibilities (a) and (b))? It must be the case that $\neg q$ is a disimplication of some other sentences in all members of W_s. That and only that would force all worlds in W_s not to include $\neg q$, and therefore to include q because they are maximal. This suggests (though lacking a test for a priori implication; I can't prove it) that (c) obtains only if S a priori implies q or some proposition that formally entails q. If this is correct, then we get the following:

A counterpossible $p > q$ is true iff one of the following obtains:

(a) q is identical to p.
(b) The relevant background facts formally imply q.
(c*) The union of p and the relevant background facts and all sentences formally implied by the relevant background facts, a priori imply q or a proposition that formally entails q, and there is a world that contains all of these sentences and includes no a priori disimplications.

5. *Where does this leave us?*

Brogaard and Salerno's account yields an implausible result. We considered an amendment, which replaces More Implications with Less Disimplications. The amended account was shown to entail a surprising result, namely that all the closest p-worlds contain *all* of the relevant background facts and any sentence formally implied by them, and that a priori implications can only play a role if such a world can be constructed with absolutely no a priori disimplications. Among other things, this means that, contrary to initial appearances, we never have to count sentences. The closest worlds cannot contain any sentences formally inconsistent with relevant background facts, nor can they contain a finite non-zero number of disimplications.

As it stands, the account is so flexible that it is difficult to falsify. So long as we lack a method for discerning what the relevant set of background facts is and what the a priori implications are in a given context, it is in principle difficult to construct a counterexample to the amended account. Suppose you have a counterpossible of the form $p > q$. Suppose the desired prediction is that this counterpossible come out true. If q is also true of the base world, then we can posit that q is a relevant background fact, and the account will give the desired prediction. If q is not true of the base world, then we can just say that it is a priori implied by p, or perhaps by the conjunction of p and some set of sentences that are true of the base world, and then we posit that those sentences from the base world are among the set of relevant background facts. Similar moves can be made if the desired result is that $p > q$ is false. There is nothing in the theory to block such ad hoc maneuvers.

To illustrate, here is how the amended account can handle the two opening examples. "If Hobbes had squared the circle, sick children in the mountains of South America at the time would not have cared" is intuitively true. In order for this to fit their account, the most natural explanation is that the consequent "sick children in the mountains of South American at the time do not care" is a relevant background fact in the context. And that's why all the nearest "Hobbes squared the circle" worlds must contain the consequent. "If intuitionistic logic were the correct logic, then the law of excluded middle would still be unrestrictedly valid" is false because in the context, "the law of excluded middle is unrestrictedly valid" is formally inconsistent with an a priori implication of the antecedent "intuitionistic logic is the correct logic". Furthermore, there exist "intuitionistic logic is the correct logic" worlds such that none of its components formally contradict any relevant background facts, nor does it consist of any a priori disimplications and all such worlds do not include "the law of excluded middle is unrestrictedly valid".

Quite generally, my view is that unfalsifiability is a reason to be less impressed by a lack of counterexamples, but it is not a reason to believe that the account is false. This flexibility does imply, however, that the amended account cannot make any substantial predictions. This is one reason to hope for a better non-vacuist account for counterpossibles. Such an account may still incorporate some of B&S's insights.

B&S call their account an "epistemic account" because their main insight is that the semantics of counterpossibles are partially determined by a priori implications. It is interesting that epistemic features rather than metaphysical features should determine the closeness relation between worlds. Perhaps this is not by chance. One of the reasons we need non-trivial counterpossibles is to allow us to evaluate propositions that are necessary if true, and impossible if false. For instance, when trying to figure out which is the correct logic, we want to know the implications of suggested logics. At least as far as this sort of need for counterpossibles is concerned, it is an epistemic need. If a certain kind of reasoning is invoked for epistemic needs, it is no surprise that epistemic factors determine the behavior of the linguistic tools we use to conduct such reasoning. My guess is that further digging into the epistemic function of counterpossibles will advance our understanding of the semantics of counterpossibles.*

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